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Referee's Report: Modélisation de systèmes dynamiques déterministes, stochastiques ou discrets. Application à l'Astronomie et la Physique.

Frederic Pierret

This is an outstanding Ph. D. thesis and serves as a model for applied mathematical research that uses fundamental mathematics, as well as modeling and computation to advance the boundaries of understanding of fundamental physical problems.

I have followed Frederic's research for several years now. The aspects of his work on stochastic dynamical systems are of particular interest to me. This is an area of increasing importance, for several reasons. Of course, "stochastic systems" are an area of great interest in pure mathematics, and "random vibrations" became a topic of great interest in engineering applications in the 1960s (and continuing to this day), but there had not been much work on the classical deterministic Hamiltonian dynamical systems from the stochastic point of view. That changed with Frederic's work on stochastic aspects of certain celestial mechanics problems. In particular, I note the following papers:

Dynamics of a rotating flat ellipsoid with a stochastic oblateness

The Sharma-Parthasarathy stochastic two-body problem

Coupling the non-gravitational forces and Modified Newton Dynamics for cometary orbits

Stochastic Gauss Equations

These are seminal papers that will serve as models for stochastic dynamics of Hamiltonian systems. By themselves, they are more than enough to be awarded the PhD degree (but they represent only a fraction of Frederic's thesis research). Like most good research, the work raises many questions for further work. In this regard, I would pose the following questions.

Question 1. Stochastic modeling is often used to model “unknown features” of a model and to “speed up” computations. In the solar system dynamics community there has been much effort in developing “long time simulations” to model the evolution of the solar system “from the beginning”. Will stochastic modeling help in developing more accurate long time simulations of the evolution of the solar system?

Question 2. The celestial mechanics models considered by Frederic are Hamiltonian. Very little of the stochastic ordinary differential equations literature has considered the particular constraints imposed on the dynamics when a symplectic structure is enforced. Is there anything “new” about “stochastic symplectic structure” and does it impose any special constraints on the dynamics that are not present in deterministic Hamiltonian systems, e.g. with respect to stability (defined in an appropriate stochastic sense)?

Question 3. There are two types of calculus typically used in the analysis of stochastic ordinary differential equations—Ito and Stratonovich. Frederic chose to work with the Stratonovich calculus. Why? Are there physical reasons for choosing Stratonovich over Ito for the applications considered? For the different stochastic dynamical systems considered in the papers above, are there differences between solutions if they are defined in the Stratonovich sense or the Ito sense/

In addition to the mathematical analyses, Frederic has also illustrated and verified his results with numerical simulations. This has resulted in the paper.

A non-standard-Euler-Maruyama scheme

The goals and performance of numerical methods for stochastic ordinary differential equations tend to be rather different when compared with deterministic ordinary differential equations. For example, Euler’s method would never be used to integrate deterministic ordinary differential equations (if one desired accuracy for “reasonable” lengths of time), but a stochastic Euler method is typically used to simulate stochastic ordinary differential equations.

Question 4. Explain why low order methods are an “acceptable choice” for integrating stochastic ordinary differential equations, but the analogous deterministic method would not be acceptable for integrating deterministic ordinary differential equations.

Question 5. For long time accurate simulation of deterministic Hamiltonian systems “symplectic integrators” are widely recognized as the method of choice. However, I am not aware that the idea of “symplectic integrators” for stochastic Hamiltonian systems has been pursued. Can Frederic comment on this? Why have they not been pursued? Are there advantages, or disadvantages?

Frederic has also worked in the area of time scale calculus. This work is described in the following paper:

Helmholtz theorem for Hamiltonian systems on time scales

Time scale calculus is an intriguing formalism developed in the late 1980s that was designed to provide a unified framework for analysing systems that exhibited aspects of both discrete and continuous time. While such problems arise frequently in applications (e.g. in control theory and robotic motion planning) it is not clear to me that this approach has been embraced by the applied community. Frederic has proven a fundamental result in this formalism—sufficient conditions for a first order system of differential equations on time scales to have a Hamiltonian structure. This result opens the door to a much more concrete, and familiar, application settings for this approach and should serve to stimulate further research in this area, both in the area of fundamental analysis, as well as in applications.

Question 6. Can you describe specific applications where this result can be used to provide new insights where traditional approaches have failed?

Frederic also has several papers on the "geometrical structure" of discretizations of continuous differential equations. This is an important area since any realistic application requires simulation on a computer and this first requires a discretization of the continuous differential equation.

Question 7. What are the outstanding issues in this topic?

In summary, this is an outstanding PhD thesis, not only for the breadth and depth of the mathematics treated, but also for the shear amount of work. It is particularly notable that essentially all of the work has been turned into papers submitted to peer reviewed journals (with several already either published or accepted).

I enthusiastically recommend that this PhD thesis be accepted and that Frederic Pierret should be awarded the degree of doctorate in mathematics.

Yours sincerely,

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