Report on the doctoral thesis of Mr Frederic Pierret:

Modélisation de systèmes dynamiques déterministes, stochastiques ou discrets. Application á l'Astronomie et la Physique

Mr Pierret has presented a very wide ranging thesis, which is motivated by problems in astronomy and physics. Essentially it is concerned with the effects of noise and numerical discretization on models of celestial dynamics in astronomy and related models in physics that conserve some quantity such as energy or momentum, and, more so, on how to include noise in such models. It draws on ideas and methods from stochastic analysis, mathematical analysis, numerical analysis as well as mechanics and dynamics, amongst others. The issues involved have already attracted much attention in the literature, as, for example, the theory of geometric integration shows. Mr Pierret is well aware of this work and other related results from a very broad range of topics. The thesis, nevertheless, is quiet distinct and insightful in its novel and original approach to the issues under consideration and also develops some very new and useful methods for their investigation.

The thesis is divided into three main parts, the first concerned with stochastic perturbations in astronomy and physics, the second with discrete structures to handle and compare dynamics on different time scales, and the third with the construction of numerical schemes, in particular for stochastic systems, which preserve certain properties or structures. Although these parts could be viewed and developed independently of each other, it is their motivation and interrelationships that form the substance of the thesis.

Ideally celestial mechanics deals with N-body problems, but these are intractable by present methods, so attention is often focused on smaller submodels such as particular 2-body models. Noise is often included to take into account the effects of the simplification and what has been neglected. In particular, since the noise-free model has a first integral form, typically a Hamiltonian structure, one would like the noisy system to have a similar property. However, the additional noise induced term in the Ito formula can effect the long term dynamical behaviour. This motivates an appropriate definition of stochastic Hamiltonian systems and methods to determine if a system is one. These issues are investigated in the thesis in terms of a stochastic version of the Sharma-Parthasarathy two-body problem, where the energy and angular momentum are shown to preserved in the weaker sense of in expectation rather than in the stronger sense for individual sample paths. More generally, the Gauss equations for variations of orbital elements are generalised to include stochastic pertubations. In addition, a toy-model for the motion of a rotating ellipsoid in the framework of stochastic differential equations is derived to model and investigate the effects of fluctuations of the shape of the earth, i.e., its geometric flattening. Numerical simulations with a suitable numerical scheme indicate similar behaviour to that obtained by experimental means. To provide a means of characterising a Hamiltonian stochastic differential equation in the Stratonvich form and to provide an appropriate Hamiltonian, Mr Pierret establishes a stochastic counterpart of a classical theorem of Helmholtz.

Numerical simulations are frequently used to obtain information about the behaviour systems modelled by differential equations. Although such schemes may converge in a technical sense, they may not preserve the long term dynamics of the system or various functional invariants or domain invariance. These issues are the focus of numerical dynamics and the more specialised theory of geometric integration, where appropriate numerical schemes are constructed. Numerical schemes can also be considered as discrete time dynamical systems and are interesting in their own right. Mr Pierret investigates both topics in the second and third parts of his thesis. In the context of numerical dynamics he constructs a novel variation of the Euler-Maruyama scheme for stochastic differential equations following the "non-standard" approach developed by R. Mickens. In particular, he uses a nonstandard function of the time step in the drift term. an error estimate is derived and strong convergence is established, and its domain invariance is shown. Moreover, although this Non-Standard Euler-Maruyama scheme is explicit like the classical Euler-Maruyama scheme, numerical simulations indicate its superior stability properties at larger stepsizes.

The second part of the thesis goes far beyond numerical dynamics. It follows from earlier work of Professor Cresson and develops a discrete embedding formalism which allows one to give a natural discrete definition of Lagrangian and Hamiltonian systems and variational integrators. Even more importantly, it allows one to explicitly compare the discrete framework with the continuous case, which is not possible with the usual discrete calculus in the literature, notably that of J.E. Marsden and his collaborators. This embedding formalism can also be used to to further develop the time-scale calculus that was introduced by S. Hilger in 1988 to unify continuous and discrete calculus. In contrast to Hilger's formalism, which used the forward and backward Euler finite difference scheme for the discrete part, in the formalism developed here the definitions are independent of the explicit terms provided by the interpolation methods. Furthermore, Pierret introduces a new framework that he calls "scale dynamics" to investigate dynamical properties of a multiscale structure, i.e., involving multiple finite and discrete time-scales. With this one can study the scale effect on the dynamics in Astronomy and Physics induced by changing the scale. It also provides a means to describe a connection between classical and quantum mechanics.

This thesis is amazing in the breadth of topics covered and the theory and skills that had to be mastered to undertake the work contained in it. It ranges from classical mechanics, through applied mathematics and mathematical analysis to stochastic analysis and introduces some very novel ideas, methods and new insights into what are essentially classical problems in astronomy and physics. The thesis is clearly written and self contained. Mr Pierret has done an excellent job and fully deserves the award of his doctorate with distinction.

Peter E. Kloeden

B.A. (Macq.), Ph.D., D.Sc. (UQld), Fellow SIAM

Professor of Applied and Instrumental Mathematics (retired)
Goethe University Frankfurt am Main Germany

Research Professor for Stochastic Dynamics
Huazhong University of Science and Technology
Wuhan China

kloeden@math.uni-frankfurt.de

10th September 2015